Comment on "Absence of electron dephasing at zero temperature"

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The recent claim by Kirkpatrick and Belitz [1] that Ward identities could be used to prove the absence of electron dephasing at $T = 0$ contains serious flaws. These authors try to draw conclusions about dephasing from an analysis of the diffuson, which is not sensitive to this process. The Cooperon, which does contain this information, is analyzed only in time reversal symmetric situations, which *by assumption* excludes any relaxation and dephasing. Hence, their analysis remains inconclusive for the problem in question.

The prediction of quantum dephasing due to electron-electron interactions at low temperature continues to attract attention. In response to our work [2] some authors have been searching for errors and/or invalid approximations in our analysis. We studied many of these – published or unpublished – claims and could discard them as being in error and/or inapplicable [2,3]. This applies also for the recently published critique by Aleiner et al. [4], to which we will respond in a separate publication.

Other authors try to prove the desired result, vanishing dephasing at $T = 0$, by general arguments. Recently Kirkpatrick and Belitz [1] (KB) argued that because of Ward identities for electrons in disordered conductors "neither a Coulomb interaction nor a short-ranged model interaction can lead to phase breaking at zero temperature in spatial dimensions $d > 2$". Here we show that KB's analysis is incorrect and inconclusive in several points:

KB do not distinguish adequately between Cooperon and diffuson. The bulk of their paper [1] is devoted to an elaborate derivation of the well known fact, that the diffuson does not decay (i.e. is massless) at $T = 0$. [Actually, as we will discuss below, also this part of KB's analysis contains serious drawbacks and, hence, cannot be accepted. However, for the moment we proceed with the observation that the diffuson is massless at $T = 0$.] The crucial question then remains whether this property of the diffuson has any implications for electron decoherence. The point is that the *Cooperon*, rather than the diffuson, contains the information about the electron dephasing time τ_φ. KB do not evaluate the Cooperon other than in "the presence of time reversal invariance". In this case the time dependences of the diffuson and the Cooperon are the same, but at the same time all relaxation and dephasing processes are excluded *by assumption.* This step constitutes KB's major error. Extending this part of KB's arguments one could "prove" that no irreversible phenomena, for instance dissipation and, of course, dephasing exist in quantum mechanics.

KB appear not to appreciate the fact that time reversal invariance is broken for an electron interacting with a bath, in the present problem with the bath produced by all other electrons [5]. This is true at any temperature, including $T = 0$, and can be observed already at the level of *exact* equations of motion [10]. When the time reversal symmetry is broken the dynamics of diffusons and Cooperons is fundamentally different. No conclusions about dephasing can be derived from the former.

KB consider primarily short-range interactions. In eq. (3.9b) of [1] we observe that KB analyze a Thomas-Fermi screened interaction rather than the *unscreened* Coulomb interaction [2]. However, the latter model [2] is generally accepted [11,12] as physically meaningful for the problem in question. The two models can lead to qualitatively differing conclusions.

For long-range interactions no meaningful results follow from the analysis of KB. One might expect that the consequences of long-range interactions could be recovered from the work of KB [1] if one sets the inverse screening length κ in eq. (3.9b) to zero. However, following their derivation to eq. (3.20) one observes that the term $W^{(dc)}$ "vanishes continuously as $\kappa \to 0$ for $d > 2$". Combining this result with eq. (3.19a) one finds that for $d > 2$ the diffuson is not affected by the interaction and does not decay at any temperature. Due to KB's assumption (3.26), this applies for the Cooperon as well. Thus, for $\kappa \to 0$ the analysis of Ref. [1] predicts the absence of electron dephasing at any temperature.

A more accurate procedure (not performed by KB) amounts to using the Fourier transform of (3.9b) $v_{sc}(q) = 4\pi e^2/(q^2 + \kappa^2)$ (in $d = 3$) rather than their eq. (3.9c). In the latter form KB expressed the strength of the interaction ($\propto e^2$) via $\kappa^2 = 4\pi e^2 N_F$ in terms of the screening length. As a result the limit $\kappa \to 0$, corresponding to long range interaction, simultaneously implies vanishing strength of the interaction. It is therefore not surprising that the diffuson is unaffected by interactions. When proceeding with the proper form of the interaction to KB's eq. (3.20) one arrives for $\kappa \to 0$ at a meaningless divergence $W^{(dc)} \propto 1/\kappa^3$. Thus, for unscreened Coulomb interaction the analysis [1] fails, and no conclusion can be drawn from it even about the diffuson.

On the other hand, the property that the diffuson does not decay at $T = 0$ *does* follow (trivially) from our formal-

ism [2,3]. Hence, KB's claim that our "calculations and arguments violate an exact Ward identity" and therefore are "not even internally consistent" lacks any substantiation and is in error.

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[1] T.R. Kirkpatrick and D. Belitz, cond-mat/0111398.

[2] D.S. Golubev and A.D. Zaikin, Phys. Rev. B **59**, 9195 (1999); Phys. Rev. B **62**, 14061 (2000).

[3] D.S. Golubev, A.D. Zaikin, and G. Schön, cond-mat/0110495.

[4] I.L. Aleiner, B.L. Altshuler, and M.G. Vavilov, cond-mat/0110545.

[5] Of course, the Hamiltonian of the *total* system of particle and bath obeys time reversal invariance. Irreversibility and dissipation appear when the bath degrees of freedom are integrated out. For further information on this issue we refer to the reviews [6–9].

[6] R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw Hill, NY, 1965), ch. 12.

[7] A.O. Caldeira, and A.J. Leggett, Ann. Phys. (N.Y.) **149**, 347 (1983).

[8] G. Schön and A.D. Zaikin, Phys. Rep. **198**, 237 (1990).

[9] U. Weiss, *Quantum Dissipative Systems*, World Scientific, Singapore, 2nd Edition (1999).

[10] See, e.g., eqs. (24), (25) in the first Ref. [2] or eq. (20) in ref. [3] where the effect of the electronic bath is expressed in terms of the fluctuating fields $V_{1,2}$.

[11] B.L. Altshuler and A.G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, eds. A.L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p.1.

[12] S. Chakravarty and A. Schmid, Phys. Rep. **140**, 193 (1986).